Montecarlo simulation to get nuclear composition using neutrino spectrum analysis

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Abstract

The neutrino Angra detector [1] is intended to become an independent monitor system of the internal fission activity of the Angra nuclear reactor. The main goal as a safeguard component is to measure the power and the evolution of the fuel composition of the reactor in real time. In the present paper we describe a simple Montecarlo simulation of the fuel composition as a function of time.

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1 Introduction

The International Atomic Energy Agency (IAEA) is the United Nations agency in charge of the development of peaceful use of atomic energy [2]. In particular IAEA is the verification authority of the Treaty on the Non-Proliferation of Nuclear Weapons (NPT). For this purpose, inspections of civil nuclear installations and related facilities under safeguards agreements are made in the participating countries. IAEA makes use of different tools for these verifications: neutron monitors; gamma spectrometers; bookkeeping of the isotopic composition of the nuclear fuel before and after their use in the nuclear reactors.

In addition, the existence of a $\bar{\nu}_e$ signal sensitive to the power and isotopic composition of a reactor core can provide the basis of another tool to address certain safeguards applications. Thus the IAEA very recently asked member states to make a feasibility study to determine whether antineutrino detection methods might provide practical safeguards tools for selected applications. If this method proves to be useful, IAEA has the power to decide that any new nuclear power plant to be built has to include an $\bar{\nu}_e$ monitor.

The high penetration power of antineutrinos and the detection capability might provide a mean to make “remote” and non-intrusive measurements of plutonium content in reactors and in large inventories of spent fuel. The antineutrino flux and energy spectrum depend on the thermal power and the fissile isotopic composition of the reactor fuel. Because the antineutrino signal from the reactor decreases as the square of the distance from the reactor to the detector, the “remote” measurement is only practical at distances of a few tens of meters if one is constrained to “small” detectors of the order of few cubic meters in size. Based on the predicted and the observed β spectra, the number of $\bar{\nu}_e$ per fission from ^{239}Pu is expected to be lower than the number from ^{235}U. This effect has been confirmed in a number of experiments [3]. This result offers a way to monitor changes in the relative amounts of ^{235}U and ^{239}Pu in the core and in freshly discharged fuel. In addition to measurements of the thermal power (including the ambient reactor temperature and the flow rate of cooling water), measurements of the antineutrino spectrum may provide an independent estimate of the isotopic composition of plutonium and uranium in the the core [4].

Detectors measuring the energy spectrum of neutrinos coming from the cores of reactors are presently used to monitor nuclear activity of reactors in Japan, USA, France and Russian, among other countries. As a first approximation, the number of neutrinos detected in a period of time is proportional to the thermal power of the reactor. However, big amounts of energy spectrum data are necessary in order to measure the isotopic composition of reactor cores because the neutrino spectrum of ^{235}U is almost identical in shape to the corresponding spectrum of other isotopes like ^{239}Pu, ^{241}Pu, etc.

In this Angra note we present a simple method to obtention the amounts of ^{235}U and ^{239}Pu in the reactor core by means of a fit of the simulated data for a given composition to a sum of the neutrino spectra as measured elsewhere [5].

2 Montecarlo Simulation

The neutrino energy spectrum for each isotope is obtained by the product of the neutrino cross section, $\sigma(E_\nu)$, and the neutrino flux, $\phi_\ell(E_\nu)$.

The neutrino flux can be computed using the following parameterized formula [6]:

$$\phi_\ell(E_\nu) = \exp\left(\sum_{k=1}^{K_\ell} a_{k\ell} E_\nu^{k-1}\right) . \tag{1}$$

The total cross section, neglecting terms of order E_ν/M, is given by the standard formula [5]:

$$\begin{aligned} \sigma_{tot}^{(0)} &= \sigma_0\ (f^2 + 3g^2)\ E_e^{(0)} p_e^{(0)} \\ &= 0.0952 \left(\frac{E_e^{(0)} p_e^{(0)}}{1\ \mathrm{MeV}^2}\right) \times 10^{-42}\ \mathrm{cm}^2 , \end{aligned} \tag{2}$$

where $E_e^{(0)} = E_\nu - (M_n - M_p)$ is the positron energy when the (small) neutron recoil is neglected, and $p_e^{(0)}$ is the corresponding momentum. The vector and axial-vector coupling constants are $f = 1, g = 1.26$ and

$$\sigma_0 = \frac{G_F^2 \cos^2\theta_C}{\pi}(1 + \Delta_{inner}^R) , \tag{3}$$

where the energy independent inner radiative corrections are $\Delta_{inner}^R \simeq 0.024$.

The resulting neutrino spectra are therefore given by the following formula [6]:

$$N_i = \frac{n_p T}{4\pi L^2} \sum_\ell N_\ell^{\mathrm{fis}} \int \mathrm{d}E_\nu\, \sigma(E_\nu)\, \phi_\ell(E_\nu)\, R_i(E_\nu) . \tag{4}$$

Here N_i is the number of neutrinos detected for each energy bin, n_p is the number of protons in the detector, L is the distance between the reactor core and the detector, and $R_i(E_\nu)$ is the detector response function for the i^{th} bin (including energy resolution and efficiencies). If the initial composition of the reactor fuel is known, the number of fissions per second N_ℓ^{fis} of each isotope ℓ can be calculated accurately (better than 1% [5]) at each burn-up stage by core simulation codes.

The Montecarlo simulation consisted in programming the neutrino spectrum (1) in such a way that it generates randomly artificial measured energies for neutrino events of detection. Three random numbers were generated in the Monte Carlo simulation to calculate each event, the first one fixes the kind of isotope, the second one establishes the neutrino energy and the third one determines if the event is within the neutrino distribution of energy.

In our calculation of composition we use two isotopes, they are ^{235}U and ^{239}Pu with a fixed arbitrary percentages of 85 and 15 per cent. The initial goal was to generate neutrino spectra with the Montecarlo simulation and to a subsequent fit to recover the core composition. The function fitted to the simulated data with the given ^{235}U and ^{239}Pu composition was:

$$S_{mixed}(E_\nu) = p0 S_{U235}(E_\nu) + p1 S_{Pu239}(E_\nu) \tag{5}$$

Here $S_{mixed}(E_\nu)$ is the spectrum function; $S_{U235}(E_\nu)$ and $S_{Pu239}(E_\nu)$ are the pure spectrums for each isotope U^{235} and Pu^{239}; p0 and p1 are the adjusting parameters resulting by a numerical fitting performed in $root$ [7].

3 Results

Once we had the Montecarlo simulation we generated mixed spectra with different number of events. The idea was to recover the fixed composition in different stages of detection. We can see in the next figures (1,2 and 3) how the calculated composition parameters p0 and p1 converge to the input values at the end with 100,000 neutrino simulated interactions.

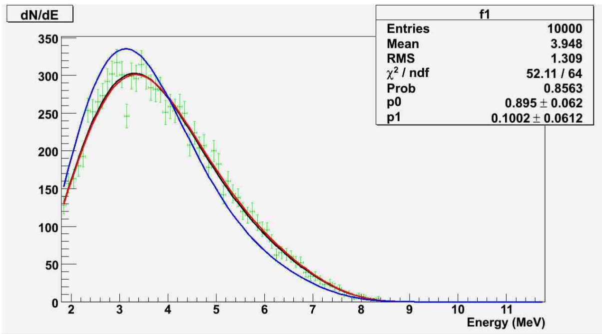

Figure 1: Fit results of Montecarlo data generated with a known composition of 85% ^{235}U and 15% ^{239}Pu using 10,000 neutrino events. The blue and red solid lines correspond to the extreme cases, 100% of ^{235}U and 100% of ^{239}Pu. The black solid line is our fitted function with fitted values of 0.895 $\pm$ 0.062 for ^{235}U and 0.1002 $\pm$ 0.0612 for ^{239}Pu.

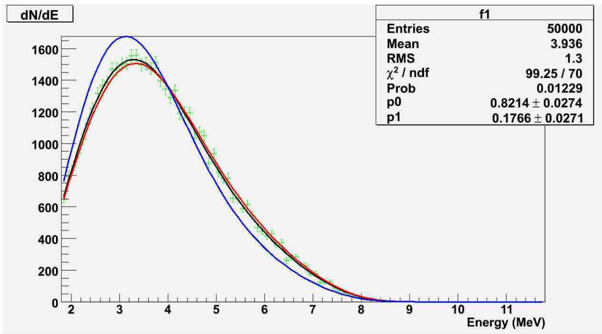

Figure 2: Fit results of Montecarlo data generated with a known composition of 85% ^{235}U and 15% ^{239}Pu using 50,000 neutrino events. The blue and red solid lines correspond to the extreme cases, 100% of ^{235}U and 100% of ^{239}Pu. The black solid line is our fitted function with fitted values of 0.8214 $\pm$ 0.0274 for ^{235}U and 0.1766 $\pm$ 0.0271 for ^{239}Pu.

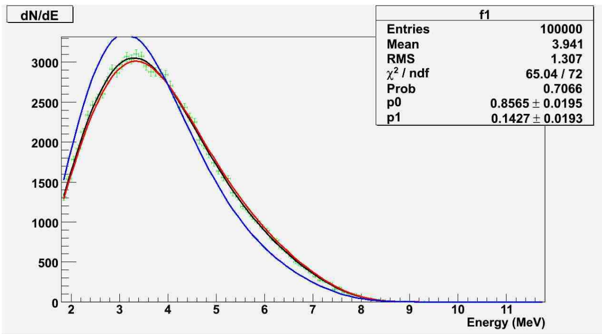

Figure 3: Fit results of Montecarlo data generated with a known composition of 85% ^{235}U and 15% ^{239}Pu using 100,000 neutrino events. The blue and red solid lines correspond to the extreme cases, 100% of ^{235}U and 100% of ^{239}Pu. The black solid line is our fitted function with fitted values of 0.8565 $\pm$ 0.0195 for ^{235}U and 0.1427 $\pm$ 0.0193 for ^{239}Pu.

These results assumed perfect measurements of the neutrino energies. If we introduce Gaussian errors on these measurements of the form dE/E = a$\sqrt{E}$, the value of p0 for ^{235}U gets higher than the real. On the other hand, the value p1 for ^{239}Pu gets lower to the corresponding real value. We can see this effect at Figure 4, where for 2% of Gaussian error in the energy measurement the obtained values were 0.8672 for ^{235}U and 0.1303 for ^{239}Pu. The more gaussian error we introduce in the detection of energy the more incorrect values for p0 and p1 we obtained. Fortunately this error is linear until 12% and consequently it is possible to rescale the parameters of composition. Figure 5 shows the over-calculated value for p0 witch is expected to be 0.85 as we increase the gaussian error in the energy detection.

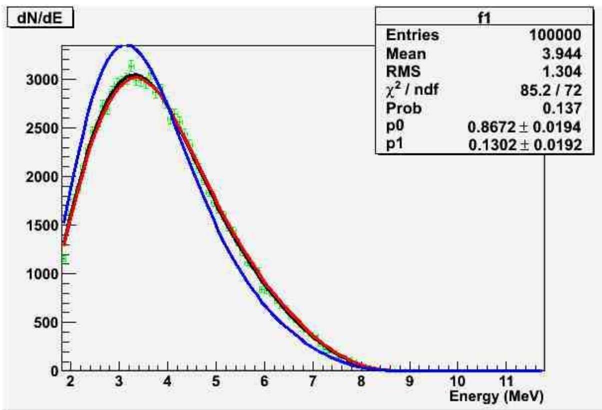

Figure 4: Fit results of Montecarlo data generated with a known composition of 85% ^{235}U and 15% ^{239}Pu using 100,000 neutrino events and setting 2% of Gaussian error in the energy measurement. The blue and red solid lines correspond to the extreme cases, 100% of ^{235}U and 100% of ^{239}Pu. The black solid line is our fitted function with fitted values of 0.8672 $\pm$ 0.0194 for ^{235}U and 0.1303 $\pm$ 0.0192 for ^{239}Pu.

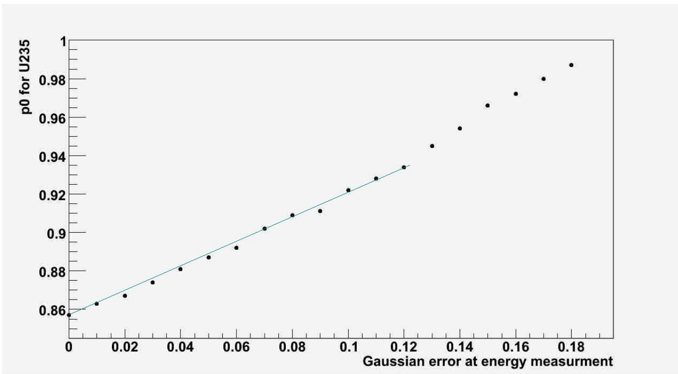

Figure 5: Over-estimation of p0 for ^{235}U as a function of the gaussian error at measuring the neutrino energy in the detector. All this points were produced as a result of generating 100,000 points.

4 Conclusions

As a final remark, it is necessary to extend this work including all the nuclear isotopes generated by the burning of ^{235}U. Furthermore, it is important to highlight the importance of getting enough neutrino interactions in the detector to calculate nuclear composition in a very short time (few days) because this process evolves gradually on time so the capabilities of size and measurement precision of the detector play the main role.

References

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